

Assignment 3

Astronomy 541

To remind you, there will not be lectures on Monday February 9 and Wednesday February 11; instead there will be lectures on Fridays February 6 and 13.

Assignment: Due Friday, February 13, in class

Here, we consider universes with matter, vacuum energy, and curvature (neglecting the role of radiation). The Hubble constant is

$$H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_R(1+z)^2 + \Omega_\Lambda}$$

where $\Omega_R = 1 - \Omega_m - \Omega_\Lambda = -kc^2/H_0^2 R_c^2$ ($k = \pm 1, 0$) includes the dynamical effects of curvature.

Problem 1 (6 pts): Produce plots of $r(z)$, the angular diameter distance, the distance modulus [$5 \log_{10}(d_L/10 \text{ pc})$], the comoving volume per square degree per unit redshift, and the age of the universe $t(z)$, all as a function of redshift (say, between 0.1 and 5). Do this for three cosmologies: $(\Omega_m, \Omega_\Lambda) = (1, 0)$, $(0.3, 0)$, and $(0.3, 0.7)$.

The last cosmology does not have analytic solutions for most of the desired quantities, so you will need to do that case (at least) numerically. Simple descriptions and codes of numerical integration can be found in *Numerical Recipes*, although for our purposes it would suffice to do the integrals as simple summations (or cumulations) with a grid size of roughly 10^{-4} in redshift (indeed, you'll get good accuracy even with 10^{-3} if you use the midpoint method). Many plotting or data manipulation packages support such cumulations. If you apply your numerical treatment to a case with a known analytic solution, you can test your accuracy.

We recommend using units of h^{-1} Mpc and h^{-1} Gyr and writing the distance modulus for $h = 1$ with an explicit $5 \log h$ remainder.

Interpret your findings, e.g. what are the trends between the cosmologies.

Problem 2 (5 pts): Imagine that we have a galaxy at $z \approx 0$ that is 10th magnitude in R at a distance of $10h^{-1}$ Mpc. If the identical galaxy were at $z = 1.75$, what would K magnitude be in each of the three cosmologies in Problem 1?

At $z = 1.75$, the light that was emitted in the R band has been shifted to the K band. Imagine that the two bands have the same fractional width, i.e. that the filter response as a function of frequency in the two bands is simply rescaled, $f_K(\nu) = f_R(2.75\nu)$. That means that the flux in the emitted R band can be easily related to that in the observed K band without knowledge of the spectrum of the object.

The flux per unit frequency of a 0th magnitude star are roughly $F_\nu = 3080$ Janskies for the R band and $F_\nu = 640$ Janskies for the K band. 1 Jansky is $10^{-23} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}$. Please note that this is flux per unit frequency; it is *not* the total flux received through the bandpass. Irrelevant note in case you're interested in the fine technical details of photometric definitions: this is the

amplitude of a spectrum with a constant flux per unit frequency that would give the same response in the detector as the calibrating $m = 0$ star.

It is almost a truism in cosmology that there is always one more factor of $1 + z$ that one has forgotten. You'll need to think carefully about what the quantities in this problem mean to be sure that your answer isn't off by a factor of 2.75!

General note: in this case, by construction, we could map the light in emitted in one band to that received in another band. In most cases, we're not this lucky: the two bands don't overlap. In this case, one has to correct the flux based on the details of the spectrum of source and the filter responses of the band passes. This is known as a K correction. These apply even if the emitted band and the observed band are the same: because the photons are redshifted, the two bands are not probing the identical part of the object's spectrum! A pedagogical explanation of K corrections is given by Hogg et al. (astro-ph/0210394).

Problem 3 (3 pts): If the number density of such galaxies is $0.01h^3 \text{ Mpc}^{-3}$ in the local universe and if the galaxies are not changing, how many of these galaxies would be predicted in a 1 square degree survey between $z = 1.7$ and $z = 1.8$? Compute this for all three cosmologies in Problem 1.

You need not do the integral between $z = 1.7$ and $z = 1.8$ more accurately than the width in redshift times the value of the integrand at the central value. Hint: This means that you can reuse the $S(r)$ calculations from Problem 2.

Problem 4 (3 pts): Consider two galaxies at $z \approx 1.75$ that are separated by $40''$ on the sky and $\Delta z = 0.003$ in redshift. Assuming that the redshift difference is strictly cosmological (probably not a good assumption as we'll learn later), what is the proper (not comoving) separation between the galaxies. Again, do this for all three cosmologies in Problem 1.

Problem 5 (3 pts): The age of a universe with $\Omega_m = 1$ and zero Λ is $2/3H_0$. We believe that the universe is at least 12 Gyr old. What does this imply about H_0 if the universe has $\Omega_m = 1$?

The formulae for the age of the universe in open ($\Lambda = 0$) models and flat models with non-zero Λ are given by Longair equations 7.31 and 7.81, reproduced below. What are the limits on H_0 is $\Omega_m = 0.2$ in these two cases?

The open model (eq. 7.31):

$$t_0 = \frac{\Omega_0}{H_0(1 - \Omega_0)^{3/2}} \left[\frac{(1 - \Omega_0)^{1/2}}{\Omega_0} - \sinh^{-1} \left(\frac{1 - \Omega_0}{\Omega_0} \right)^{1/2} \right] \quad (1)$$

The flat Λ model (eq. 7.81):

$$t_0 = \frac{2}{3H_0\Omega_\Lambda^{1/2}} \ln \left[\frac{1 + \Omega_\Lambda^{1/2}}{(1 - \Omega_\Lambda)^{1/2}} \right] \quad (2)$$

Needless to say, the number of good back-of-the-envelope calculations that you can now do is very large, so spend some time putting these formulae to work in different astrophysical contexts!