

Assignment 2

Astronomy 541

To remind you, there will not be lectures on Monday February 9 and Wednesday February 11; instead there will be lectures on Fridays February 6 and 13.

First, some **ungraded** back-of-the-envelope calculations to try:

1) How many stars are there in a seeing-limited pixel (say, square arcsecond) in a typical location in the Andromeda galaxy (700 kpc distant)? In a galaxy in the Virgo cluster (18 Mpc distant)? If you know about the technique of distance determination by surface brightness fluctuations, consider the implications of this calculations.

2) With plausible detection limits in the optical, how far away can one detect RR Lyrae or horizontal branch stars ($M \approx 0.5$)? Cepheid variables ($M \approx -4$)? Globular clusters ($M \approx -8$)? Luminous novae ($M \approx -9$)?

Assignment: Due Wednesday, February 4, in class

In this assignment, we will write the Robertson-Walker metric as

$$ds^2 = c^2 dt^2 - R(t)^2 \left[dr^2 + S(r)^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] \quad (1)$$

where $R(t)$ is the expansion factor and

$$S(r) = \begin{cases} R_c \sin(r/R_c) & \text{for the spherical geometry} \\ r & \text{for the flat geometry} \\ R_c \sinh(r/R_c) & \text{for the hyperbolic geometry} \end{cases}$$

with R_c being a constant. We will use the convention that $R = 1$ today, so the redshift is $1 + z = 1/R(t)$. We will assume that $R(t)$ is monotonically increasing with time, so the mapping between redshift and time is one-to-one.

Lengths measured by

$$d\ell^2 = \left[dr^2 + S(r)^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]$$

are known as comoving lengths, whereas those lengths multiplied by $R(t)$ are the physical length at some time (i.e. the length that would be measured by a meter stick).

The Hubble constant (defined as the ratio between physical velocities and physical distances) as a function of time is defined as

$$H(t) = \frac{1}{R(t)} \frac{dR}{dt}$$

Problem 1 (7 pts): Using the above as a starting point, consider a flat cosmology in which the scale factor $R(t) = (t/t_0)^{2/3}$ where t_0 is the age of the universe today. We will later find that this is the case of a critical density, matter-dominated universe. Compute:

a) The Hubble constant as a function of redshift.

b) The comoving distance (also known as the coordinate distance) between us and a given redshift.

c) The angular diameter distance at a given redshift, i.e. the radians subtended on the sky by an object spanning one unit of length transverse to our line-of-sight.

d) The luminosity distance at a given redshift, i.e. the distance needed to convert bolometric luminosity to bolometric flux.

e) The comoving volume per unit redshift per steradian at a given redshift.

f) The elapsed time between a given redshift and today.

Also, although it is not part of the problem set, be sure that you can reproduce the derivation of the formulae for the above starting from the FRW metric.

Problem 2 (4 pts): Show that for any geometry the comoving volume over the whole sky interior to a given redshift is

$$V(< z) = 4\pi \int_0^{r(z)} dr S(r)^2.$$

Begin from the formula for the comoving volume per redshift and solid angle

$$\frac{dV}{dz d\Omega} = \frac{c}{H(z)} S^2(r(z)). \quad (2)$$

Problem 3 (4 pts): We claimed in class that particles propagating in an expanding universe have their momenta decrease in proportion to R^{-1} . For a non-relativistic particle (that is, velocity much less than the speed of light), show that this is equivalent to a particle moving inertially while its velocity is being measured relative to an expanding set of observers. The velocity/momentum in question is that being measured by the observer co-located with the particle at each instant of time; in other words, it's a different observer at each time.

What does inertially mean? It would be tempting to say that the particle continues to move at velocity v_0 in the rest-frame of some observer A. However, this is not correct because the space between the particle and observer A at later times is expanding. For example, imagine that the scale factor $R(t)$ is constant for a while so that the particle moves some distance away. Now let the scale factor increase very rapidly. According to observer A, the particle (and the comoving observers near it) now has an enormous velocity, not with which the v_0 it started.

So inertial motion is only defined relative to nearby observers. You'll need to consider small steps and then build the global result by adding up many of them.